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International Journal of Heat and Mass Transfer 47 (2004) 4491–4495
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Technical Note

On the natural convection of water near its density inversion in an inclined square cavity

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Received 17 July 2003; received in revised form 27 May 2004

Abstract

Experimental and numerical results of the natural convection of water in an inclined square cavity at temperatures near its maximum density, as published in the literature [J. Fluid Mech. 142 (1984) 363; Int. J. Heat Mass Transfer 30 (1987) 2319], are revisited. These results were thoughtfully reviewed and some inconsistencies were found. The same physical problem is considered in this technical note, using the spectral elements method. Close agreement is obtained with the numerical results of both previous reports and corroborate the observations in [Int. J. Heat Mass Transfer 30 (1987) 2319] about the substantial differences between the numerical results and the experimental data presented in [J. Fluid Mech. 142 (1984) 363] for the temperature profiles.

Keywords: Natural convection; Inclined square cavity; Water density inversion; Spectral methods

1. Introduction

Experimental and numerical results of the natural convection of water in an inclined square cavity at temperatures near its maximum density, have been published in the literature [1]. Starting from a heated-from-below condition, these authors presented flow patterns, temperature distributions and the average heat-transfer coefficient for various temperature differences between the hot and cold walls and inclination angles of the cavity. These results have been compared with a numerical solution [2], although only temperature profiles and average heat-transfer coefficient were considered. Good agreement was found with the results in [1] for the average heat transfer, but substantial differences with the temperature profiles. The authors of [2] pointed out that the temperature profiles numerically obtained in [1] have the same shape as one would expect for a unicellular Boussinesq flow, however they do not coincide with the dual-cell structure experimentally found by the same referred authors. In the present technical communication, the same physical problem is considered, this time using a high-order numerical procedure: the spectral elements method. The present results are in close agreement with the numerical results of both previous reports and corroborate the observations in [2] about the substantial differences between the numerical results and the experimental data presented in [1] for the temperature profiles.

2. Description of the problem

Consider a square cavity with two adiabatic side vertical walls, filled with cold water in the density inversion range. The bottom surface of the cavity is heated from below at constant, uniform temperature T_h, while the opposite top surface is kept at T_c. The cavity has an inclination angle θ, with respect to the vertical direction.

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doi:10.1016/j.ijheatmasstransfer.2004.06.003

Nomenclature

c_p	specific heat (J/kg K)
g	gravitational acceleration (m/s^2)
L	length of the cavity (m)
p	pressure (N/m^2)
$Pr = \frac{\alpha}{v}$	Prandtl number
$Ra = \frac{gL^3}{v\alpha}$	Rayleigh number
T	temperature (°C)
u, v	velocity components (m/s)
x, y	Cartesian coordinates (m)
X^*, Y^*	nondimensional coordinates, x/L, y/L

Greek symbols

α	thermal diffusivity (m^2/s)
θ	angle of inclination of the cavity (°)
v	kinematic viscosity (m^2/s)

Subscripts

c	cold wall
h	hot wall
0	reference value

The steady-state mathematical model of the problem makes use of the governing equations (continuity, momentum and energy) and the boundary conditions, entirely the same way as Inaba and Fukuda [1]. However, the governing equations in the present investigation, considered all physical properties of water (density, thermal conductivity, specific heat and viscosity) as functions of temperature, through polynomials of either, second or fourth degree [3]. This kind of correlations were used by Inaba and Fukuda [1] only for the density. The following equations and boundary conditions are therefore considered:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + v\nabla^2 u \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + v\nabla^2 v - \frac{\rho}{\rho_0}g \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\nabla^2 T \tag{4}$$

and the corresponding boundary conditions are:

$$T(x,0) = T_h, \quad T(x,L) = T_c \tag{5}$$

$$\frac{\partial T}{\partial x}(0,y) = \frac{\partial T}{\partial x}(L,y) = 0 \tag{6}$$

$$u = v = 0 \text{ at all boundaries.} \tag{7}$$

3. Numerical method

The spectral elements method (SEM), is a high-order, weighted-residual technique that combines the generality and geometric flexibility of low-order, h-type finite element methods, with the rapid convergence rate of the spectral methods. It was introduced by Patera [4] and has been used previously in some studies of Fluid Dynamics [4–6]. For the spectral element discretization, the computational domain is broken up into macro-elements and each element is isoparametrically mapped from the physical space to the local coordinate system. The geometry and the pressure, velocity and temperature fields are represented as a tensor product of high-order Lagrangian interpolants through Gauss–Lobatto collocation points. A wide review of this method has been presented in [7]. Since one of the main advantages of SEM is to achieve the convergence to the exact solution by increasing the degree of the polynomial expansions, while keeping the number of macro-elements fixed, the mesh-independence analysis was carried out with a fixed number of macro-elements (100), while they were used different numbers of collocation points. In this way, a computational mesh of 10×10 macro-elements, was used, each macro-element with 49 Gauss–Lobatto interpolation points to ensure solution convergence.

Steady-state natural convection of water ($Pr = 13$, $Ra = 1.03 \times 10^7$) in a square cavity (side length of $L = 0.015$ m) heated from below, with two adiabatic side vertical walls, was considered. The temperature at the heated bottom ($y = 0$) was altered in the density inversion range (2 °C < T_h < 14 °C), while the temperature T_c at the cold ceiling remained in 0 °C. The simulation was carried out for two inclination angles of the cavity: $\theta = 30°$ and 90°. These physical properties, dimensions and boundary conditions were therefore, the same as in Inaba and Fukuda [1].

4. Results and discussion

Fig. 1 shows the steady-state flow and temperature fields for $\theta = 90°$, with $T_h = 8$ °C and $T_c = 0°$. The flow patterns in Fig. 1a, clearly describe two symmetrical cells rotating in opposite directions, as expected for this physical situation where the fluid temperature range is centered in the density inversion point.

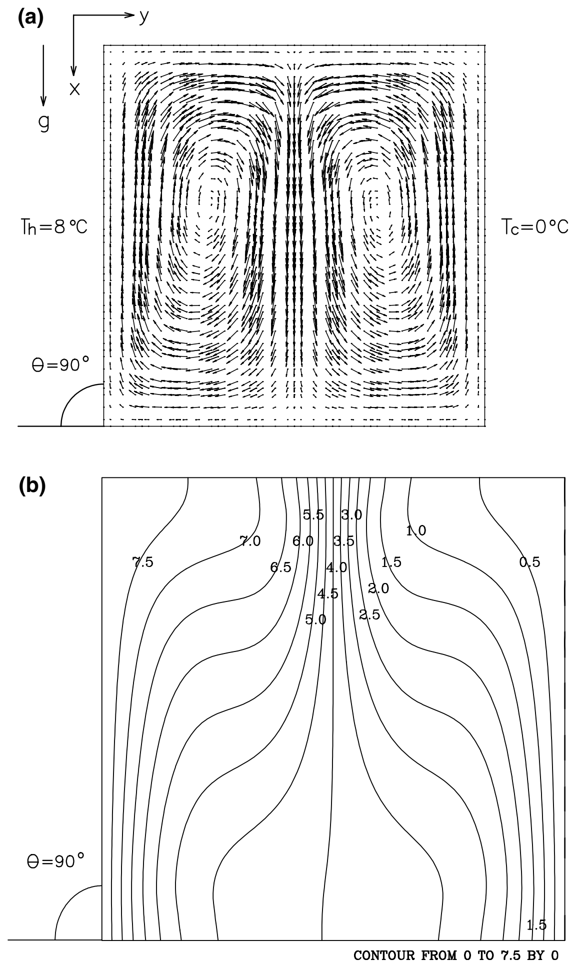

Fig. 1. (a) Flow pattern and (b) temperature field in an inclined cavity for $\theta = 90°$, $T_h = 8$ °C, $Ra_H = 1.03 \times 10^7$, $Pr = 13$.

In Fig. 2 on the other hand, the dimensionless temperature distribution at the middle of the cavity ($X^* = x/L = 0.5$) (Fig. 2a) and the average Nusselt number as a function of T_h (Fig. 2b) obtained in the present investigation, are compared with the experimental data of Inaba and Fukuda [1], and the numerical predictions by Lin and Nansteel [2], for $\theta = 90°$. It can be observed that there is a fair agreement between the present results and the experimental data for the average Nusselt number. Nevertheless, there is also a significant discrepancy for the temperature distribution. Note that the experimental data of Inaba and Fukuda [1, Fig. 11a, p. 376], repeated here in Fig. 2a, suggest a profile more typical of a unicellular flow, with maximum temperature gradients near the isothermal walls and a single central region where the temperature is almost uniform. This is not the expected flow pattern for the physical problem herein considered. The temperature distribution obtained in the present investigation—in agreement with those predicted by Lin and Nansteel—clearly imply the two symmetrical cells rotating in opposite directions showed in Fig. 1. This was also noticed by the latter authors, [2].

Fig. 3 shows the flow pattern and temperature field for the cavity at $\theta = 30°$ and $T_h = 8$ °C. The double-cell structure (one big cell occupying a large portion area of

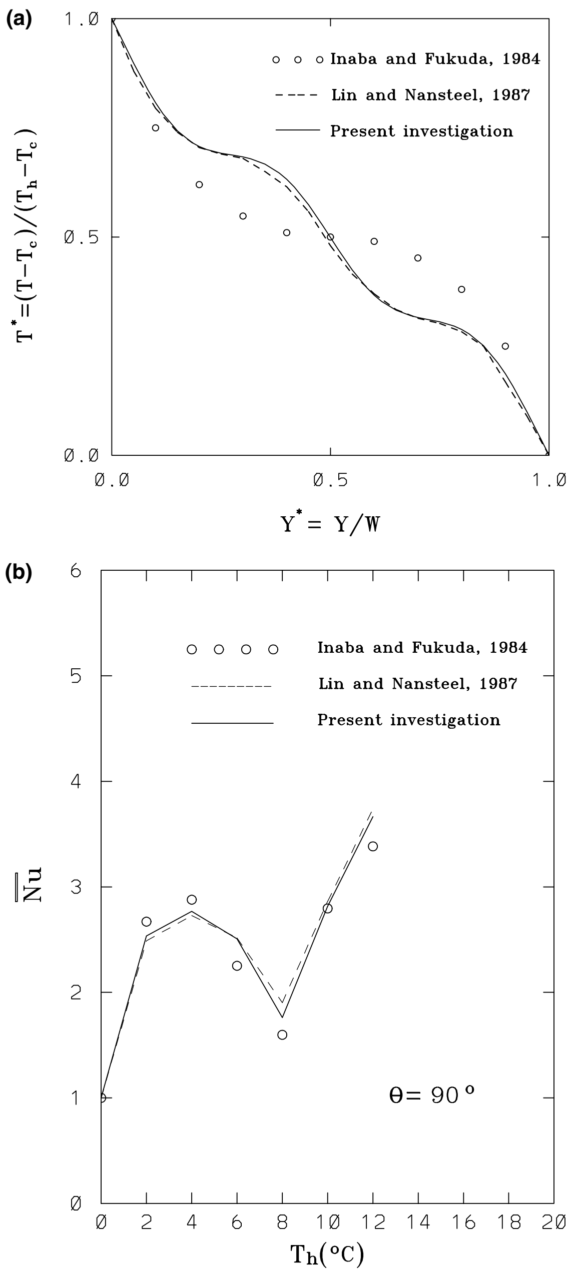

Fig. 2. (a) Temperature distribution in the middle plane of the cavity ($T_h = 8$ °C) and (b) average Nusselt number vs. T_h in an inclined cavity for $\theta = 90°$, $Ra_H = 1.03 \times 10^7$, $Pr = 13$.

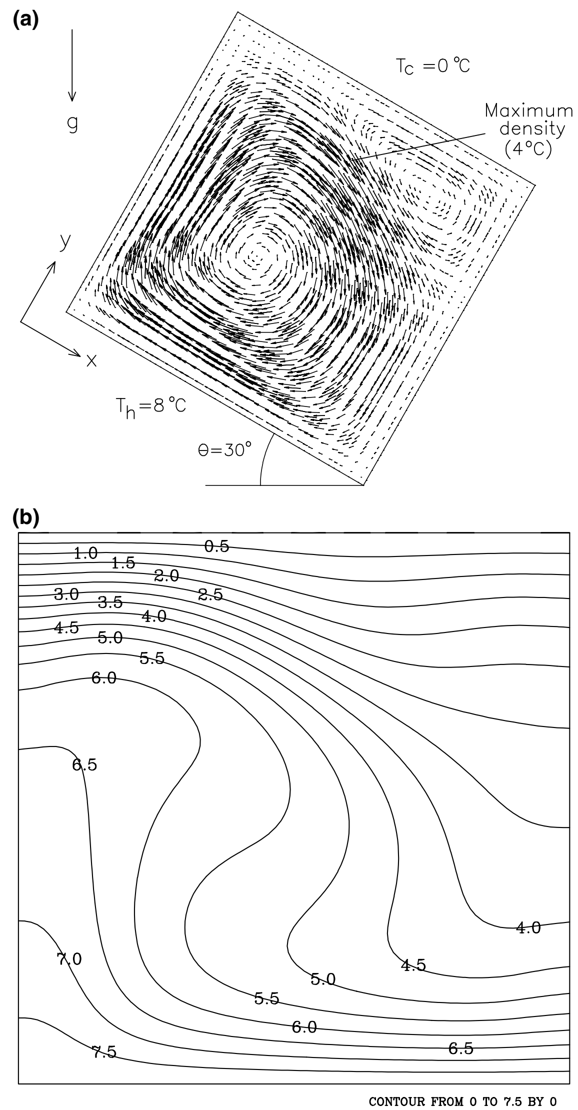

Fig. 3. (a) Flow pattern and (b) temperature field in an inclined cavity for $\theta = 30°$, $T_h = 8$ °C, $Ra_H = 1.03 \times 10^7$, $Pr = 13$.

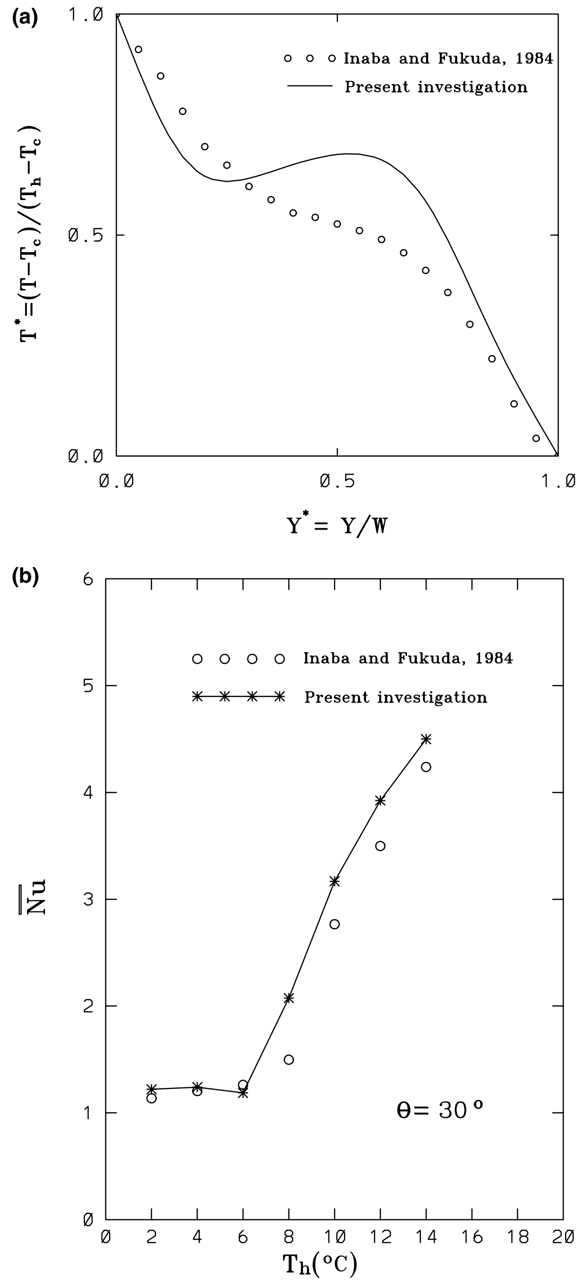

Fig. 4. (a) Temperature distribution in the middle plane of the cavity ($T_h = 8$ °C) and (b) average Nusselt number vs. T_h in an inclined cavity for $\theta = 30°$, $Ra_H = 1.03 \times 10^7$, $Pr = 13$.

the cavity, and a small cell in the upper right corner) is corroborated in this figure. The shape and size of the natural convection cells are strongly influenced by the angle of inclination. This flow pattern (Fig. 3a) is qualitatively similar to the ones photographed and reported by Inaba and Fukuda [1, Fig. 7a, p. 372], for similar conditions.

Fig. 4 on the other hand, shows the temperature profile at the middle section of the cavity ($X^* = x/L = 0.5$) and the average Nusselt number for $\theta = 30°$. The present results are compared with those in [1]. As can be seen, the temperature field obtained in this investigation, is what one would expect for this physical situation, but disagrees and shows the inconsistency of the experimental results obtained by Inaba and Fukuda [1]. According to the present investigation, the maximum density fluid (corresponding to 4 °C) is located at the region where the two cells merge together (indicated by an arrow in Fig. 3a). This result is consistent in Figs.

3a,b and 4a. However, it is discrepant with the results of Inaba and Fukuda [1] as they reported the central region of the cavity for this maximum density flow [Fig. 10a, p. 376], repeated in Fig. 4 of this paper for comparison purposes. This is a most unlikely and unexpected result. Moreover, this result corresponds again to a unicellular flow pattern. This confirms the inconsistency of the experimental results obtained by Inaba and Fukuda [1].

5. Concluding remarks

Numerical simulation of natural convection of water in an inclined square cavity at temperatures near density inversion were carried out using spectral elements method. The results for the flow patterns, temperature distributions and average heat-transfer coefficient, were consistent and expected. They agree well with numerical results reported in the literature, Lin and Nansteel [2]. They confirmed though, the inconsistencies of the experimental results of Inaba and Fukuda [1].

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